

РАЦИОНАЛНИ АЛГЕБАРСКИ ИЗРАЗИ

(први део)

1. Упростити изразе:

а) $x^6 : x^2 = \boxed{x^4}$

б) $(x^6)^2 = \boxed{x^{12}}$

в) $x^6 \cdot x^2 = \boxed{x^8}$

г) $x^6 : x^6 = x^0 = \boxed{1}$

д) $x^6 : x^5 = x^1 = \boxed{x}$

ђ) $x^6 : x = \boxed{x^5}$

е) $x^6 \cdot x = \boxed{x^7}$

$a^m \cdot a^n = a^{m+n}$
 $a^m : a^n = \frac{a^m}{a^n} = a^{m-n}$
 $(a^m)^n = a^{m \cdot n}$
 $a^0 = 1$ $a^1 = a$

2. Упростити изразе:

а) $x^{20} : (x^7 \cdot x^4) = x^{20} : x^{11} = \boxed{x^9}$

б) $(x^4)^3 \cdot (x^5 : x^2)^3 = x^{12} \cdot (x^3)^3 = x^{12} \cdot x^9 = \boxed{x^{21}}$

в) $-x^{11} : (x^8 : x^3)^2 = -x^{11} : (x^5)^2 = -x^{11} : x^{10} = -x^1 = \boxed{-x}$

г) $\frac{x^9 \cdot x^5 : (x^2)^2}{x^8 : x^2} = \frac{x^{14} : x^4}{x^6} = \frac{x^{10}}{x^6} = \boxed{x^4}$

д) $\frac{(x^3)^2 \cdot x^4 : (x^2)^2}{x^6 : (x^4 : x^2)^2} = \frac{x^6 \cdot x^4 : x^4}{x^6 : (x^2)^2} = \frac{x^{10} : x^4}{x^6 : x^4} = \frac{x^6}{x^2} = \boxed{x^4}$

3. Упрости изразе:

а) $(2xy^2)^2 \cdot (2x^2y) = (2^2 x^2 (y^2)^2) \cdot (2x^2y) = 4x^2y^4 \cdot 2x^2y = \boxed{8x^4y^5}$

б) $(2xy^2)^2 : (2x^2y) = (4x^2y^4) : (2x^2y) = 2x^0y^3 = \boxed{2y^3}$

в) $\frac{(5a^2b^3)^3 \cdot a^4b^5}{25a^7b^{12}} = \frac{125a^6b^9 \cdot a^4b^5}{25a^7b^{12}} = \frac{125a^{10}b^{14}}{25a^7b^{12}} = \boxed{5a^3b^2}$

г) $\frac{(2x^3y^2)^4}{8x^5y^3xy^5} = \frac{2^4(x^3)^4 \cdot (y^2)^4}{8x^5y^8} = \frac{16x^{12}y^8}{8x^5y^8} = 2x^6y^0 = \boxed{2x^6}$

$$д) \frac{(4ab^3)^4 \cdot a^4 b^5}{16a^7 b^{11}} = \frac{256 a^4 b^{12} \cdot a^4 b^5}{16a^7 b^{11}} = \frac{\overset{16}{256} a^8 b^{17}}{\underset{1}{16} a^7 b^{11}} = 16 a^1 b^6 = \boxed{16ab^6}$$

4. Израчунај:

$$а) 5 - (-5)^2 - 5^3 = 5 - 25 - 125 = -20 - 125 = \boxed{-145}$$

$$б) 5^4 \cdot (-5)^7 : (5^3)^3 = 5^4 \cdot (-5^7) : 5^9 = -5^{11} : 5^9 = -5^2 = \boxed{-25}$$

$$в) 3 - (-3)^2 - 3^3 = 3 - 9 - 27 = -6 - 27 = \boxed{-33}$$

$$г) 3^4 \cdot (-3)^6 : 3^8 = 3^4 \cdot 3^6 : 3^8 = 3^{10} : 3^8 = 3^2 = \boxed{9}$$

ИЗЛОЖИЛАЧ НЕПАРАН
 $(-5)^7 = -5^7$

ИЗЛОЖИЛАЧ ПАРАН
 $(-3)^6 = 3^6$

5. Израчунај вредност израза

$$а) \frac{2^6 \cdot 4^2}{2^2} = \frac{2^6 \cdot (2^2)^2}{2^2} = \frac{2^6 \cdot 2^4}{2^2} = \frac{2^2}{2^2} = 2^0 = \boxed{1}$$

$$б) \frac{2^5 \cdot 4^3}{8^2} = \frac{2^5 \cdot (2^2)^3}{(2^3)^2} = \frac{2^5 \cdot 2^6}{2^6} = \frac{2^{11}}{2^6} = 2^5 = \boxed{32}$$

$$в) \frac{4^2 \cdot 16^2}{4^6} = \frac{4^2 \cdot (4^2)^2}{4^6} = \frac{4^2 \cdot 4^4}{4^6} = \frac{4^6}{4^6} = 4^0 = \boxed{1}$$

$$г) \frac{5^6 \cdot 25^4}{5^8} = \frac{5^6 \cdot (5^2)^4}{5^8} = \frac{5^6 \cdot 5^8}{5^8} = 5^6 = 125 \cdot 125 = \boxed{15625}$$

$$д) \frac{16^4 \cdot 2^{16}}{4^{16}} = \frac{(2^4)^4 \cdot 2^{16}}{(2^2)^{16}} = \frac{2^{16} \cdot 2^{16}}{2^{32}} = \frac{2^{32}}{2^{32}} = 2^0 = \boxed{1}$$

$$е) \frac{3^7 \cdot 9^3}{27^3} = \frac{3^7 \cdot (3^2)^3}{(3^3)^3} = \frac{3^7 \cdot 3^6}{3^9} = \frac{3^{13}}{3^9} = 3^4 = \boxed{81}$$

Израчунај вредност израза

6.

$$а) \frac{(-2)^{10} \cdot 2^5 \cdot 2^{12}}{(2^8)^3} = \frac{2^{10} \cdot 2^5 \cdot 2^{12}}{2^{24}} = \frac{2^{27}}{2^{24}} = 2^3 = \boxed{8}$$

$$б) \frac{2^4 \cdot 2^{15} \cdot (-2)^8}{(2^8)^3} = \frac{2^4 \cdot 2^{15} \cdot 2^8}{2^{24}} = \frac{2^{27}}{2^{24}} = 2^3 = \boxed{8}$$

7. Израчунај вредност израза

$$а) (2^2)^2 + (3^2)^2 - 5^2 = (4)^2 + 9^2 - 25 = 16 + 81 - 25 = 97 - 25 = \boxed{72}$$

$$б) 5^2 - (2^3)^2 - (2^2)^2 = 25 - 8^2 - 4^2 = 25 - 64 - 16 = -39 - 16 = \boxed{-55}$$

$$в) (2^2)^3 - (-3^2)^2 + 5^2 = 4^3 - (-9)^2 + 25 = 64 - 81 + 25 = -17 + 25 = \boxed{8}$$

$$г) (3^2)^2 - (-2^2)^3 - 5^2 = 9^2 - (-4)^3 - 25 = 81 - (-64) - 25 = 81 + 64 - 25 = 145 - 25 = \boxed{120}$$

Изračунај вредност израза

8.

$$а) \left(\frac{1}{2}\right)^3 \cdot \left(\frac{16}{3}\right)^3 \cdot \left(\frac{3}{8}\right)^3 = \left(\frac{1}{2} \cdot \frac{16}{3} \cdot \frac{3}{8}\right)^3 = 1^3 = \boxed{1}$$

$$б) \left(\frac{3}{5}\right)^5 \cdot \left(\frac{2}{9}\right)^5 \cdot \left(7\frac{1}{2}\right)^5 = \left(\frac{3}{5} \cdot \frac{2}{9} \cdot \frac{15}{2}\right)^5 = 1^5 = \boxed{1}$$

$$в) \left(1\frac{1}{7}\right)^8 \cdot \left(1\frac{3}{4}\right)^8 : 2^8 = \left(\frac{8}{7} \cdot \frac{7}{4} : 2\right)^8 = (2 : 2)^8 = 1^8 = \boxed{1}$$

$$г) 4^4 \cdot \left(1\frac{1}{2}\right)^4 \cdot \left(\frac{1}{3}\right)^4 = \left(4 \cdot \frac{3}{2} \cdot \frac{1}{3}\right)^4 = 2^4 = \boxed{16}$$

Изračунај вредност израза

9.

$$а) (2^2)^3 + \frac{(-4)^3}{2^4} - \frac{8^2 \cdot 2^3}{4^2} = 4^3 + \frac{-64}{16} - \frac{64 \cdot 8}{16} = 64 - 4 - 32 = 60 - 32 = \boxed{28}$$

$$б) (-2^3)^2 + \frac{2^5}{4^2} - \frac{8^2 \cdot (-2)^3}{2^4} = (-8)^2 + \frac{32}{16} - \frac{64 \cdot (-8)}{16} = 64 + 2 + 32 = 66 + 32 = \boxed{98}$$

10. Изračунај бројевну вредност датих израза за $x = -1$

$$а) 3x^2 - 4x + 5 = 3 \cdot (-1)^2 - 4 \cdot (-1) + 5 = 3 \cdot 1 + 4 + 5 = 3 + 4 + 5 = \boxed{12}$$

$$б) -4x^3 - 3x^2 + 4x + 5 = -4 \cdot (-1)^3 - 3 \cdot (-1)^2 + 4 \cdot (-1) + 5 = -4 \cdot (-1) - 3 \cdot 1 + (-4) + 5 = 4 - 3 - 4 + 5 = \boxed{2}$$

$$в) 2x^4 - 4x^3 - 3x^2 + 4x + 5 = 2 \cdot (-1)^4 - 4 \cdot (-1)^3 - 3 \cdot (-1)^2 + 4 \cdot (-1) + 5 = 2 \cdot 1 - 4 \cdot (-1) - 3 \cdot 1 - 4 + 5 = 2 + 4 - 3 - 4 + 5 = \boxed{4}$$

11. Изračунај бројевну вредност израза $\frac{2a^3b^3}{5a+b^2}$ за $a = -1, b = 2$.

$$\frac{2 \cdot (-1)^3 \cdot 2^3}{5 \cdot (-1) + 2^2} = \frac{2 \cdot (-1) \cdot 8}{-5 + 4} = \frac{-16}{-1} = \boxed{16}$$

12. Изračунај бројевну вредност израза $\frac{3a^2+2b^3}{5a^2b}$ за $a = 1, b = -2$.

$$\frac{3 \cdot 1^2 + 2 \cdot (-2)^3}{5 \cdot 1^2 \cdot (-2)} = \frac{3 \cdot 1 + 2 \cdot (-8)}{5 \cdot 1 \cdot (-2)} = \frac{3 - 16}{-10} = \frac{-13}{-10} = \boxed{\frac{13}{10}}$$

13. За $a = -2$, $b = 3$, $c = -4$ израчунај бројевну вредност израза:

a) $a^4 - b^3 + c^2 = (-2)^4 - 3^3 + (-4)^2 = 16 - 27 + 16 = 32 - 27 = \boxed{5}$

б) $3a^2 - 2b + 4c^3 = 3 \cdot (-2)^2 - 2 \cdot 3 + 4 \cdot (-4)^3 = 3 \cdot 4 - 6 + 4 \cdot (-64) = 12 - 6 - 256 = 6 - 256 = \boxed{-250}$

в) $\frac{1}{2}a^3 - \frac{2}{3}b^3 + \frac{1}{4}c^4 = \frac{1}{2} \cdot (-2)^3 - \frac{2}{3} \cdot 3^3 + \frac{1}{4} \cdot (-4)^4 = \frac{1}{2} \cdot (-8) - \frac{2}{3} \cdot 27 + \frac{1}{4} \cdot 256 = -4 - 18 + 64 = -22 + 64 = \boxed{42}$

г) $(4a + 3b + c)^4 = (4 \cdot (-2) + 3 \cdot 3 + (-4))^4 = (-8 + 9 - 4)^4 = (-3)^4 = \boxed{81}$

14. Израчунај бројевну вредност израза :

a) $\frac{x^2 + 2x - 1}{x - 1}$ за $x = -\frac{1}{2}$; $\frac{(-\frac{1}{2})^2 + 2 \cdot (-\frac{1}{2}) - 1}{-\frac{1}{2} - 1} = \frac{\frac{1}{4} - 1 - 1}{-\frac{1}{2} - \frac{2}{2}} = \frac{\frac{1}{4} - 2}{-\frac{3}{2}} = \frac{\frac{1}{4} - \frac{8}{4}}{-\frac{3}{2}} = \frac{-\frac{7}{4}}{-\frac{3}{2}} = \frac{-7}{4} \cdot \frac{2}{-3} = \frac{7}{6} = \boxed{\frac{7}{6}}$

б) $\frac{x^3 - x^2 + 2}{x^2 - 2}$ за $x = -2$; $\frac{(-2)^3 - (-2)^2 + 2}{(-2)^2 - 2} = \frac{-8 - 4 + 2}{4 - 2} = \frac{-12 + 2}{2} = \frac{-10}{2} = \boxed{-5}$

в) $\frac{2 - x^3}{x^4 - 1}$ за $x = \frac{1}{2}$; $\frac{2 - (\frac{1}{2})^3}{(\frac{1}{2})^4 - 1} = \frac{2 - \frac{1}{8}}{\frac{1}{16} - 1} = \frac{\frac{16}{8} - \frac{1}{8}}{\frac{1}{16} - \frac{16}{16}} = \frac{\frac{15}{8}}{-\frac{15}{16}} = -\frac{15 \cdot 16}{8 \cdot 15} = -\frac{16}{8} = \boxed{-2}$

г) $\frac{x^4 - 2x^2 + 4}{x^2 - 1}$ за $x = \sqrt{2}$; $\frac{(\sqrt{2})^4 - 2 \cdot (\sqrt{2})^2 + 4}{(\sqrt{2})^2 - 1} = \frac{4 - 2 \cdot 2 + 4}{2 - 1} = \frac{4 - 2 + 4}{1} = \frac{4}{1} = \boxed{4}$
 $(\sqrt{2})^4 = ((\sqrt{2})^2)^2 = 2^2 = 4$

15. Израчунај бројевну вредност израза ако је $x = 5$, $y = 0$ и $z = -2$.

a) $2 \cdot (x - y^3 z^2)$; $2 \cdot (5 - 0^3 \cdot (-2)^2) = 2 \cdot (5 - 0 \cdot 4) = 2 \cdot (5 - 0) = 2 \cdot 5 = \boxed{10}$

б) $2 \cdot (x - y^3) \cdot z^2$; $2 \cdot (5 - 0^3) \cdot (-2)^2 = 2 \cdot (5 - 0) \cdot 4 = 2 \cdot 5 \cdot 4 = \boxed{40}$

в) $2 \cdot (x - y^3 z)^2$; $2 \cdot (5 - 0^3 \cdot (-2))^2 = 2 \cdot (5 - 0 \cdot (-2))^2 = 2 \cdot (5 - 0)^2 = 2 \cdot 5^2 = 2 \cdot 25 = \boxed{50}$

г) $2 \cdot (x - y)^3 \cdot z^2$; $2 \cdot (5 - 0)^3 \cdot (-2)^2 = 2 \cdot 5^3 \cdot 4 = 2 \cdot 125 \cdot 4 = \boxed{1000}$