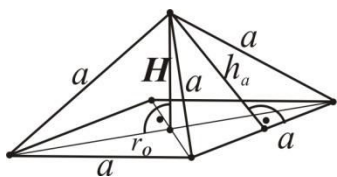


Правилна једнакоивична четворострана пирамида



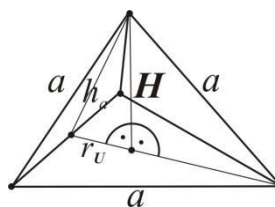
$$B = a^2$$

$$M = 4 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$M = a^2 \sqrt{3}$$

$$h_a = \frac{a\sqrt{3}}{2}$$

Правилна једнакоивична тространа пирамида - ТЕТРАЕДАР



$$B = \frac{a^2 \sqrt{3}}{4}$$

$$M = 3 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$P = 4 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$P = a^2 \sqrt{3}$$

$$h_a = \frac{a\sqrt{3}}{2}$$

1. Израчунај површину правилне једнакоивичне четворостране пирамиде ако је основна ивица 8cm

$$a = 8\text{cm}$$

$$P, V = ?$$

$$B = a^2$$

$$B = 8^2$$

$$B = 64\text{cm}^2$$

$$M = 4 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$M = a^2 \sqrt{3}$$

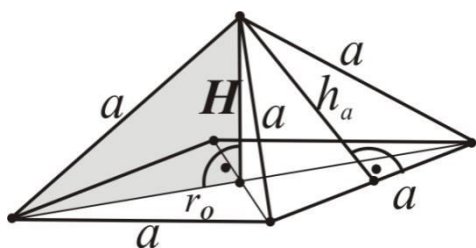
$$M = 8^2 \sqrt{3}$$

$$M = 64\sqrt{3}\text{cm}^2$$

$$P = B + M$$

$$P = 64 + 64\sqrt{3}$$

$$P = 64(1 + \sqrt{3})\text{cm}^2$$



$$d = a\sqrt{2}$$

$$d = 8\sqrt{2}\text{cm}$$

$$r_o = \frac{d}{2}$$

$$r_o = \frac{8\sqrt{2}}{2}$$

$$r_o = 4\sqrt{2}\text{cm}$$

$$s^2 = H^2 + r_o^2$$

$$a^2 = H^2 + r_o^2$$

$$8^2 = H^2 + (4\sqrt{2})^2$$

$$64 = H^2 + 16 \cdot 2$$

$$64 = H^2 + 32$$

$$H^2 = 64 - 32$$

$$H^2 = 32$$

$$H = \sqrt{32}$$

$$H = \sqrt{16 \cdot 2}$$

$$H = 4\sqrt{2}\text{cm}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 64 \cdot 4\sqrt{2}$$

$$V = \frac{256\sqrt{2}}{3}\text{cm}^3$$

2. Израчунај површину и запремину правилне једнакоивичне тростране пирамиде ако је основна ивица 6cm

$$a = 6\text{cm}$$

$$P, V = ?$$

$$P = 4 \cdot \frac{a^2 \cdot \sqrt{3}}{4}$$

$$P = a^2 \sqrt{3}$$

$$P = 6^2 \sqrt{3}$$

$$P = 36\sqrt{3}\text{cm}^2$$

$$B = \frac{a^2 \sqrt{3}}{4}$$

$$r_o = \frac{a\sqrt{3}}{3}$$

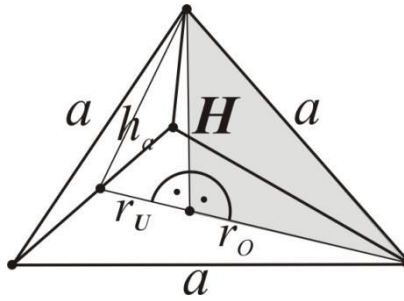
$$B = \frac{6^2 \sqrt{3}}{4}$$

$$r_o = \frac{6\sqrt{3}}{3}$$

$$B = \frac{36\sqrt{3}}{4}$$

$$r_o = 2\sqrt{3}\text{cm}$$

$$B = 9\sqrt{3}\text{cm}^2$$



$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 9\sqrt{3} \cdot 2\sqrt{6}$$

$$V = 6\sqrt{18}$$

$$V = 6\sqrt{9 \cdot 2}$$

$$V = 6 \cdot 3\sqrt{2}$$

$$V = 18\sqrt{2}\text{cm}^3$$

$$s^2 = H^2 + r_o^2$$

$$a^2 = H^2 + r_o^2$$

$$6^2 = H^2 + (2\sqrt{3})^2$$

$$36 = H^2 + 4 \cdot 3$$

$$36 = H^2 + 12$$

$$H^2 = 36 - 12$$

$$H^2 = 24$$

$$H = \sqrt{24}$$

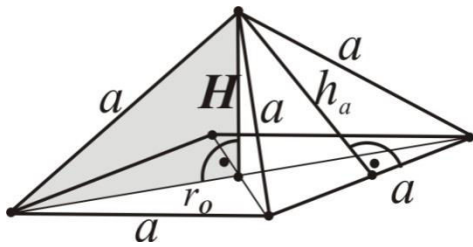
$$H = \sqrt{4 \cdot 6}$$

$$H = 2\sqrt{6}\text{cm}$$

3. Израчунај запремину једнакоивичне четворостране пирамиде чија је основна ивица 12cm .

$$a = 12\text{cm}$$

$$V = ?$$



$$d = a\sqrt{2}$$

$$d = 12\sqrt{2}\text{cm}$$

$$r_o = \frac{d}{2}$$

$$r_o = \frac{12\sqrt{2}}{2}$$

$$r_o = 6\sqrt{2}\text{cm}$$

$$B = a^2$$

$$B = 12^2$$

$$B = 144\text{cm}^2$$

$$s^2 = H^2 + r_o^2$$

$$a^2 = H^2 + r_o^2$$

$$12^2 = H^2 + (6\sqrt{2})^2$$

$$144 = H^2 + 36 \cdot 2$$

$$144 = H^2 + 72$$

$$H^2 = 144 - 72$$

$$H^2 = 72$$

$$H = \sqrt{72}$$

$$H = \sqrt{36 \cdot 2}$$

$$H = 6\sqrt{2}\text{cm}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 144 \cdot 6\sqrt{2}$$

$$V = 288\sqrt{2}\text{cm}^3$$

4. Израчунај запремину тетраедра чија је површина $144\sqrt{3}cm^2$.

$$\frac{P = 144\sqrt{3}cm^2}{V = ?}$$

$$P = 4 \cdot \frac{a^2\sqrt{3}}{4}$$

$$144\sqrt{3} = a^2\sqrt{3}$$

$$a^2 = \frac{144\sqrt{3}}{\sqrt{3}}$$

$$a^2 = 144$$

$$a = \sqrt{144}$$

$$\boxed{a = 12cm}$$

$$r_o = \frac{a\sqrt{3}}{3}$$

$$r_o = \frac{12\sqrt{3}}{3}$$

$$\boxed{r_o = 4\sqrt{3}cm}$$

$$a^2 = H^2 + r_o^2$$

$$12^2 = H^2 + (4\sqrt{3})^2$$

$$144 = H^2 + 48$$

$$H^2 = 144 - 48$$

$$H^2 = 96$$

$$H = \sqrt{96}$$

$$H = \sqrt{16 \cdot 6}$$

$$\boxed{H = 4\sqrt{6}cm}$$

$$B = \frac{a^2\sqrt{3}}{4}$$

$$B = \frac{12^2\sqrt{3}}{4}$$

$$B = \frac{144\sqrt{3}}{4}$$

$$\boxed{B = 36\sqrt{3}cm^2}$$

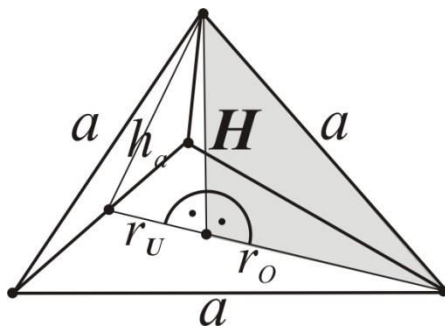
$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 36\sqrt{3} \cdot 4\sqrt{6}$$

$$V = 48\sqrt{18}$$

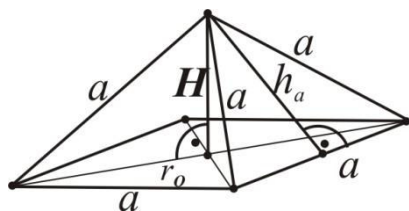
$$V = 48 \cdot 3\sqrt{2}$$

$$\boxed{V = 144\sqrt{2}cm^3}$$



5. Израчунај површину pravilne једнакоивичне четворостране пирамиде ако је апотема $12cm$.

$$\frac{h_a = 12cm}{P = ?}$$



$$h_a = \frac{a\sqrt{3}}{2}$$

$$12 = \frac{a\sqrt{3}}{2}$$

$$a\sqrt{3} = 24$$

$$a = \frac{24}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$a = \frac{24\sqrt{3}}{3}$$

$$\boxed{a = 8\sqrt{3}cm}$$

$$B = a^2$$

$$B = (8\sqrt{3})^2$$

$$B = 64 \cdot 3$$

$$\boxed{B = 192cm^2}$$

$$M = 4 \cdot \frac{a^2\sqrt{3}}{4}$$

$$M = a^2\sqrt{3}$$

$$M = (8\sqrt{3})^2 \sqrt{3}$$

$$M = 64 \cdot 3\sqrt{3}$$

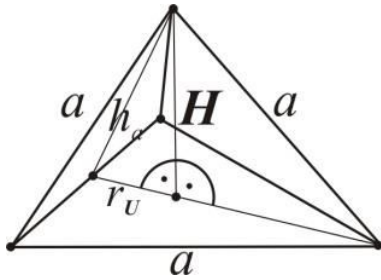
$$\boxed{M = 192\sqrt{3}cm^2}$$

$$P = B + M$$

$$P = 192 + 192\sqrt{3}$$

$$\boxed{P = 192(1 + \sqrt{3})cm^2}$$

6. Апотема једнакоивичне троугране пирамиде је $6\sqrt{3}cm$. Израчунај запремину те пирамиде.



$$h_a = \frac{a\sqrt{3}}{2}$$

$$r_o = \frac{a\sqrt{3}}{3}$$

$$a^2 = H^2 + r_o^2$$

$$B = \frac{a^2\sqrt{3}}{4}$$

$$9\sqrt{3} = \frac{a\sqrt{3}}{2}$$

$$r_o = \frac{18\sqrt{3}}{3}$$

$$18^2 = H^2 + (6\sqrt{3})^2$$

$$B = \frac{18^2\sqrt{3}}{4}$$

$$a\sqrt{3} = 18\sqrt{3}$$

$$\boxed{r_o = 6\sqrt{3}cm}$$

$$324 = H^2 + 108$$

$$B = \frac{324\sqrt{3}}{4}$$

$$a = \frac{18\sqrt{3}}{\sqrt{3}}$$

$$H^2 = 324 - 108$$

$$H^2 = 216$$

$$H^2 = 216$$

$$H = \sqrt{216}$$

$$\boxed{B = 81\sqrt{3}cm^2}$$

$$\boxed{a = 18cm}$$

$$\boxed{H = 6\sqrt{6}cm}$$

$$\begin{array}{r|l} 216 & 2 \\ 108 & 2 \\ 54 & 2 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$216 = 2^2 \cdot 3^2 \cdot 2 \cdot 3$$

$$\sqrt{216} = \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{2} \cdot \sqrt{3}$$

$$\sqrt{216} = 2 \cdot 3 \cdot \sqrt{2} \cdot \sqrt{3}$$

$$\sqrt{216} = 6 \cdot \sqrt{6}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 81\sqrt{3} \cdot 6\sqrt{6}$$

$$V = 162\sqrt{18}$$

$$V = 162 \cdot 3\sqrt{2}$$

$$\boxed{V = 486\sqrt{2}cm^3}$$

7. Израчунај дужину основне ивице тетраедра ако је површина њеног омотача износи $48\sqrt{3}cm^2$

$$\frac{M = 48\sqrt{3}cm^2}{a = ?}$$

$$M = 3 \cdot \frac{a^2\sqrt{3}}{4} \quad a^2 = 64$$

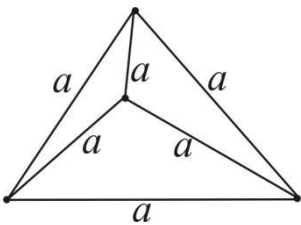
$$a = \sqrt{64}$$

$$48\sqrt{3} = 3 \cdot \frac{a^2\sqrt{3}}{4} \quad \boxed{a = 8cm}$$

$$\frac{48\sqrt{3}}{1} = \frac{3 \cdot a^2\sqrt{3}}{4}$$

$$3a^2\sqrt{3} = 4 \cdot 48\sqrt{3}$$

$$a^2 = \frac{4 \cdot 48 \sqrt{3}}{\cancel{3}^1 \sqrt{3}}$$



8. Израчунај дужину основне ивице једнакоивичне четворостране пирамиде ако је површина њеног омотача износи $100\sqrt{3}cm^2$.

$$\frac{M = 100\sqrt{3}cm^2}{a = ?}$$

$$M = 4 \cdot \frac{a^2\sqrt{3}}{4}$$

$$M = a^2\sqrt{3}$$

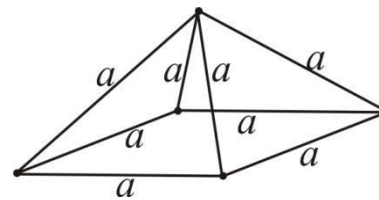
$$100\sqrt{3} = a^2\sqrt{3}$$

$$a^2 = \frac{100\sqrt{3}}{\sqrt{3}}$$

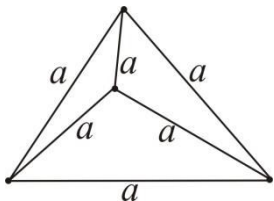
$$a^2 = 100$$

$$a = \sqrt{100}$$

$$\boxed{a = 10cm}$$



9. Израчунај дужину основне ивице тетраедра ако збир дужина свих ивица тетраедра износи $30 cm$.

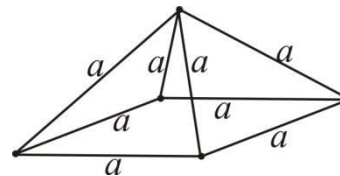


$$6 \cdot a = 30cm$$

$$a = \frac{30}{6}$$

$$\boxed{a = 5cm}$$

10. Израчунај дужину основне ивице правилне четворостране пирамиде ако збир дужина свих њених ивица износи $32 cm$.



$$8 \cdot a = 32cm$$

$$a = \frac{32}{8}$$

$$\boxed{a = 4cm}$$

11. Површина омотача правилне једнакоивичне чртворостране пирамиде је $36\sqrt{3}cm^2$.
Одреди запремину те пирамиде.

$$M = 36\sqrt{3}cm^2$$

$$V = ?$$

$$M = 4 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$M = a^2 \sqrt{3}$$

$$36\sqrt{3} = a^2 \sqrt{3}$$

$$a^2 = 36$$

$$a^2 = 36$$

$$a = \sqrt{36}$$

$$\boxed{a = 6cm}$$

$$d = a\sqrt{2}$$

$$\boxed{d = 6\sqrt{2}cm}$$

$$r_o = \frac{d}{2}$$

$$r_o = \frac{6\sqrt{2}}{2}$$

$$\boxed{r_o = 3\sqrt{2}cm}$$

$$B = a^2$$

$$B = 6^2$$

$$\boxed{B = 36cm^2}$$

$$s^2 = H^2 + r_o^2$$

$$a^2 = H^2 + r_o^2$$

$$6^2 = H^2 + (3\sqrt{2})^2$$

$$36 = H^2 + 9 \cdot 2$$

$$36 = H^2 + 18$$

$$H^2 = 36 - 18$$

$$H^2 = 18$$

$$H = \sqrt{18}$$

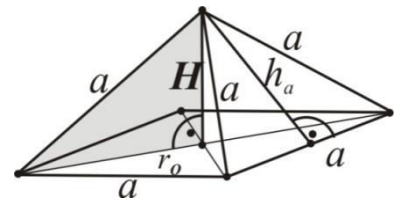
$$H = \sqrt{9 \cdot 2}$$

$$\boxed{H = 3\sqrt{2}cm}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 36 \cdot 3\sqrt{2}$$

$$\boxed{V = 36\sqrt{2}cm^3}$$



12. Одреди запремину једнакоивичне тростране пирамиде чија је површина за $12\sqrt{3}cm^2$ већа од површине њене основе.

$$P = B + 12\sqrt{3}cm^2$$

$$V = ?$$

$$P = B + 12\sqrt{3}$$

$$B + M = B + 12\sqrt{3}$$

$$\boxed{M = 12\sqrt{3}cm^2}$$

$$M = 3 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$12\sqrt{3} = 3 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$\frac{12\sqrt{3}}{1} = \frac{3 \cdot a^2 \sqrt{3}}{4}$$

$$3a^2 \sqrt{3} = 4 \cdot 12\sqrt{3}$$

$$a^2 = \frac{4 \cdot 12^4 \sqrt{3}}{\cancel{3}^1 \sqrt{3}}$$

$$a^2 = 16$$

$$a = \sqrt{16}$$

$$\boxed{a = 4cm}$$

$$r_o = \frac{a\sqrt{3}}{3}$$

$$\boxed{r_o = \frac{4\sqrt{3}}{3}cm}$$

$$a^2 = H^2 + r_o^2$$

$$4^2 = H^2 + \left(\frac{4\sqrt{3}}{3}\right)^2$$

$$16 = H^2 + \frac{16 \cdot 3}{9}$$

$$16 = H^2 + \frac{16}{3} / \cdot 3$$

$$48 = 3H^2 + 16$$

$$3H^2 = 48 - 16$$

$$3H^2 = 32$$

$$H^2 = \frac{32}{3}$$

$$\boxed{H = \sqrt{\frac{32}{3}}cm}$$

$$B = \frac{a^2 \sqrt{3}}{4}$$

$$B = \frac{4^2 \sqrt{3}}{4}$$

$$B = \frac{16\sqrt{3}}{4}$$

$$\boxed{B = 4\sqrt{3}cm^2}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot 4\sqrt{3} \cdot \sqrt{\frac{32}{3}}$$

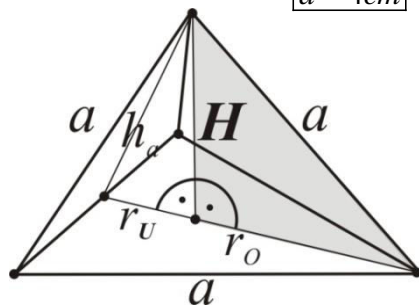
$$V = \frac{4}{3} \cdot \sqrt{3 \cdot \frac{32}{3}}$$

$$V = \frac{4}{3} \cdot \sqrt{32}$$

$$V = \frac{4}{3} \cdot \sqrt{16 \cdot 2}$$

$$V = \frac{4}{3} \cdot 4\sqrt{2}$$

$$\boxed{V = \frac{16}{3}\sqrt{2}cm^3}$$



13. Израчунај површину правилне једнакоивичне четворостране пирамиде ако јевисина пирамиде 6 cm.

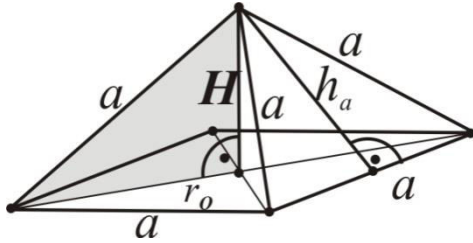
ПРВИ НАЧИН:

$$H = 6 \text{ cm}$$

$$P = ?$$

$$r_o = \frac{d}{2}$$

$$r_o = \frac{a\sqrt{2}}{2}$$



$$s^2 = H^2 + r_o^2$$

$$a^2 = 6^2 + \left(\frac{a\sqrt{2}}{2}\right)^2$$

$$a^2 = 36 + \frac{a^2 \cdot 2}{4}$$

$$a^2 = 36 + \frac{a^2}{2} / \cdot 2$$

$$2a^2 = 72 + a^2$$

$$2a^2 - a^2 = 72$$

$$a^2 = 72$$

$$a = \sqrt{72}$$

$$a = \sqrt{36 \cdot 2}$$

$$a = 6\sqrt{2} \text{ cm}$$

$$B = a^2$$

$$B = (6\sqrt{2})^2$$

$$B = 36 \cdot 2$$

$$B = 72 \text{ cm}^2$$

$$M = 4 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$M = a^2 \sqrt{3}$$

$$M = (6\sqrt{2})^2 \sqrt{3}$$

$$M = 36 \cdot 2 \sqrt{3}$$

$$M = 72\sqrt{3} \text{ cm}^2$$

$$P = B + M$$

$$P = (72 + 72\sqrt{3}) \text{ cm}^2$$

$$P = 72(1 + \sqrt{3}) \text{ cm}^2$$

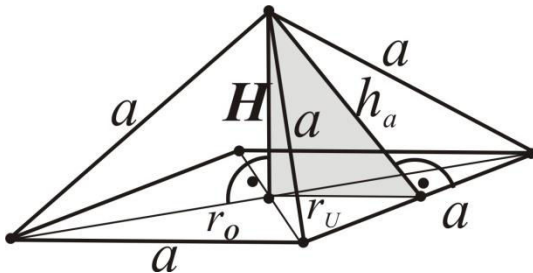
ДРУГИ НАЧИН:

$$H = 8 \text{ cm}$$

$$P = ?$$

$$h_a = \frac{a\sqrt{3}}{3}$$

$$r_u = \frac{a}{2}$$



$$h_a^2 = H^2 + r_u^2$$

$$\left(\frac{a\sqrt{3}}{3}\right)^2 = 6^2 + \left(\frac{a}{2}\right)^2$$

$$\frac{a^2 \cdot 3}{4} = 36 + \frac{a^2}{4}$$

$$\frac{3a^2}{4} = 36 + \frac{a^2}{4} / \cdot 4$$

$$3a^2 = 144 + a^2$$

$$3a^2 - a^2 = 144$$

$$2a^2 = 144$$

$$a^2 = \frac{144}{2}$$

$$a^2 = 72$$

$$a = \sqrt{72}$$

$$a = \sqrt{36 \cdot 2}$$

$$a = 6\sqrt{2} \text{ cm}$$

$$B = a^2$$

$$B = (6\sqrt{2})^2$$

$$B = 36 \cdot 2$$

$$B = 72 \text{ cm}^2$$

$$M = 4 \cdot \frac{a^2 \sqrt{3}}{4}$$

$$M = a^2 \sqrt{3}$$

$$M = (6\sqrt{2})^2 \sqrt{3}$$

$$M = 36 \cdot 2 \sqrt{3}$$

$$M = 72\sqrt{3} \text{ cm}^2$$

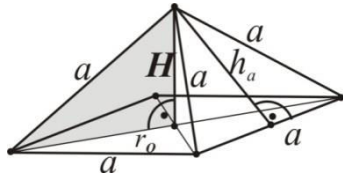
$$P = B + M$$

$$P = (72 + 72\sqrt{3}) \text{ cm}^2$$

$$P = 72(1 + \sqrt{3}) \text{ cm}^2$$

14.* Израчунај основну ивицу правилне једнакоивичне четворостране пирамиде ако је њена запремина $36\sqrt{2}cm^3$.

$$\frac{V = 36\sqrt{2}cm^3}{a = ?}$$



$$s^2 = H^2 + r_o^2$$

$$a^2 = H^2 + \left(\frac{a\sqrt{2}}{2}\right)^2$$

$$a^2 = H^2 + \frac{a^2 \cdot 2}{4}$$

$$a^2 = H^2 + \frac{a^2}{2} \cdot 2$$

$$2a^2 = 2H^2 + a^2$$

$$2H^2 = 2a^2 - a^2$$

$$2H^2 = a^2$$

$$H^2 = \frac{a^2}{2}$$

$$H = \sqrt{\frac{a^2}{2}}$$

$$H = \frac{a}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{H = \frac{a\sqrt{2}}{2}}$$

$$\boxed{B = a^2}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot a^2 \cdot \frac{a\sqrt{2}}{2}$$

$$36\sqrt{2} = \frac{a^3\sqrt{2}}{6}$$

$$a^3\sqrt{2} = 36\sqrt{2} \cdot 6$$

$$a^3\sqrt{2} = 216\sqrt{2}$$

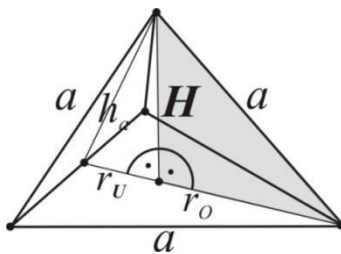
$$a^3 = \frac{216\sqrt{2}}{\sqrt{2}}$$

$$a^3 = 216$$

$$\boxed{a = 6cm}$$

15.* Израчунај основну ивицу тетраедра ако је његова запремина $\frac{16\sqrt{2}}{3}cm^3$.

$$\frac{V = \frac{16\sqrt{2}}{3}cm^3}{a = ?}$$



$$s^2 = H^2 + r_o^2$$

$$a^2 = H^2 + \left(\frac{a\sqrt{3}}{3}\right)^2$$

$$a^2 = H^2 + \frac{a^2 \cdot 3}{9}$$

$$a^2 = H^2 + \frac{a^2}{3} \cdot 3$$

$$3a^2 = 3H^2 + a^2$$

$$3H^2 = 3a^2 - a^2$$

$$3H^2 = 2a^2$$

$$H^2 = \frac{2a^2}{3}$$

$$H = \sqrt{\frac{2a^2}{3}}$$

$$H = \frac{a\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{H = \frac{a\sqrt{6}}{3}}$$

$$\boxed{B = \frac{a^2\sqrt{3}}{4}}$$

$$V = \frac{1}{3} \cdot B \cdot H$$

$$V = \frac{1}{3} \cdot \frac{a^2\sqrt{3}}{4} \cdot \frac{a\sqrt{6}}{3}$$

$$\frac{16\sqrt{2}}{3} = \frac{a^3\sqrt{18}}{36}$$

$$\frac{16\sqrt{2}}{3} = \frac{a^3 \cdot 3\sqrt{2}}{36}$$

$$\frac{16\sqrt{2}}{3} = \frac{a^3\sqrt{2}}{12}$$

$$3a^3\sqrt{2} = 16\sqrt{2} \cdot 12$$

$$a^3 = \frac{16\sqrt{2} \cdot 12}{3\sqrt{2}}$$

$$a^3 = 64$$

$$\boxed{a = 4cm}$$

