

РАДНИ ЛИСТ
РАЦИОНАЛНИ АЛГЕБАРСКИ ИЗРАЗИ
(други део)

1. Одреди квадрат бинома:

<p>КВАДРАТ ЗБИРА</p> $(I + II)^2 = I^2 + 2 \cdot I \cdot II + II^2$

<p>КВАДРАТ РАЗЛИКЕ</p> $(I - II)^2 = I^2 - 2 \cdot I \cdot II + II^2$

$$1) \underbrace{(x)}_I - \underbrace{(5)}_II = x^2 - 2 \cdot x \cdot 5 + 5^2 = \boxed{x^2 - 10x + 25}$$

$$2) \underbrace{(y)}_I + \underbrace{(3)}_II = y^2 + 2 \cdot y \cdot 3 + 3^2 = \boxed{y^2 + 6y + 9}$$

$$3) \underbrace{(2z)}_I - \underbrace{(1)}_II = (2z)^2 - 2 \cdot 2z \cdot 1 + 1^2 = \boxed{4z^2 - 4z + 1}$$

$$4) \underbrace{(2)}_I - \underbrace{(3a)}_II = 2^2 - 2 \cdot 2 \cdot 3a + (3a)^2 = \boxed{4 - 12a + 9a^2}$$

$$5) \underbrace{(-b)}_I + \underbrace{(3)}_II = (-b)^2 + 2 \cdot (-b) \cdot 3 + 3^2 = \boxed{b^2 - 6b + 9}$$

$$6) \underbrace{(-2t)}_I - \underbrace{(3)}_II = (-2t)^2 - 2 \cdot (-2t) \cdot 3 + 3^2 = \boxed{4t^2 + 12t + 9}$$

$$7) \underbrace{\left(x + \frac{1}{2}\right)}_I = x^2 + 2 \cdot x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = \boxed{x^2 + x + \frac{1}{4}}$$

$$8) \underbrace{\left(-\frac{3}{4}x + \frac{1}{2}\right)}_I = \left(-\frac{3}{4}x\right)^2 + 2 \cdot \left(-\frac{3}{4}x\right) \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = \boxed{\frac{9}{16}x^2 - \frac{3}{4}x + \frac{1}{4}}$$

$$9) \underbrace{\left(-\frac{2}{5}x - \frac{3}{5}\right)}_I = \left(-\frac{2}{5}x\right)^2 + 2 \cdot \left(-\frac{2}{5}x\right) \cdot \frac{3}{5} + \left(\frac{3}{5}\right)^2 = \boxed{\frac{4}{25}x^2 - \frac{12}{25}x + \frac{9}{25}}$$

$$10) \underbrace{(0,1x + 0,5)}_I = (0,1x)^2 + 2 \cdot 0,1x \cdot 0,5 + (0,5)^2 = \boxed{0,01x^2 + 0,1x + 0,25}$$

$$11) \underbrace{(-0,2x + 0,5)}_I = (-0,2x)^2 + 2 \cdot (-0,2x) \cdot 0,5 + 0,5^2 = \boxed{0,04x^2 - 0,2x + 0,25}$$

$$12) \underbrace{(2 - 1,5y)}_I = 2^2 - 2 \cdot 2 \cdot 1,5y + (1,5y)^2 = \boxed{4 - 6y + 2,25y^2}$$

$$13) \underbrace{(-0,2x - 1,5)}_I = (-0,2x)^2 - 2 \cdot (-0,2x) \cdot 1,5 + 1,5^2 = \boxed{0,04x^2 + 0,6x + 2,25}$$

$$14) \underbrace{(3x)}_I + \underbrace{(5y)}_II)^2 = (3x)^2 + 2 \cdot 3x \cdot 5y + (5y)^2 = 9x^2 + 30xy + 25y^2$$

$$15) \underbrace{(2a)}_I - \underbrace{(5b)}_II)^2 = (2a)^2 + 2 \cdot 2a \cdot 5b + (5b)^2 = 4a^2 + 20ab + 25b^2$$

$$16) \underbrace{(2a^2)}_I + \underbrace{(3b^3)}_II)^2 = (2a^2)^2 + 2 \cdot 2a^2 \cdot 3b^3 + (3b^3)^2 = 4a^4 + 12a^2b^3 + 9b^6$$

$$17) \underbrace{(-c^4)}_I + \underbrace{(4b^3)}_II)^2 = (-c^4)^2 + 2 \cdot (-c^4) \cdot 4b^3 + (4b^3)^2 = c^8 - 8c^4b^3 + 16b^6$$

$$18) \underbrace{(-3x^3)}_I - \underbrace{(2y^2)}_II)^2 = (-3x^3)^2 - 2 \cdot (-3x^3) \cdot (2y^2) + (2y^2)^2 = 9x^6 + 12x^3y^2 + 4y^4$$

$$19) \underbrace{(3x^4y)}_I + \underbrace{(2x^2y)}_II)^2 = (3x^4y)^2 + 2 \cdot 3x^4y \cdot 2x^2y + (2x^2y)^2 = 9x^8y^2 + 12x^6y^2 + 4x^4y^2$$

$$20) \underbrace{(1)}_I - \underbrace{(2x^2y^2)}_II)^2 = 1^2 - 2 \cdot 1 \cdot 2x^2y^2 + (2x^2y^2)^2 = 1 - 4x^2y^2 + 4x^4y^4$$

$$21) \left(\underbrace{\frac{1}{2}xy}_I - \underbrace{0,2x^2}_{\frac{1}{5}x^2} \right)^2 = \left(\frac{1}{2}xy \right)^2 - 2 \cdot \frac{1}{2}xy \cdot \frac{1}{5}x^2 + \left(\frac{1}{5}x^2 \right)^2 = \frac{1}{4}x^2y^2 - \frac{1}{5}x^3y + \frac{1}{25}x^4$$

2. Упрости изразе:

1) $x + x = 2x$ 2) $x \cdot x = x^2$ 3) $x^2 \cdot x^2 = x^4$ 4) $x^2 + x^2 = 2x^2$

5) $-2x \cdot 5x^2y = -10x^3y$ 6) $-7ab^3 \cdot 4a^3b = -28a^4b^4$ 7) $(2a^2b^3)^2 = 4a^4b^6$

8) $4x + 5x = 9x$ 9) $4x \cdot 5x = 20x^2$ 10) $-7x^4y \cdot 44y = -308x^4y^2$

11) $-3a^4b^2 \cdot 2a^3b = -6a^7b^3$ 12) $(2a^4b^2)^3 = 8a^{12}b^6$

13) $2x \cdot (x^2 + 5x - 3) = 2x^3 + 10x^2 - 6x$

14) $5x^4 \cdot (x^2 - 3x + 1) = 5x^6 - 15x^5 + 5x^4$

$(a \cdot b)^n = a^n \cdot b^n$
 $a^n \cdot a^m = a^{n+m}$

$$15) (x-3) \cdot (x-2) = x^2 - 2x - 3x + 6 = \boxed{x^2 - 5x + 6}$$

$$16) (x+4) \cdot (x-2) = x^2 - 2x + 4x - 8 = \boxed{x^2 + 2x - 8}$$

3. Коришћењем формуле за разлику квадрата, среди изразе:

$$1) (x^2 + 1) \cdot (x-1) \cdot (x+1) = (x^2 + 1) \cdot (x^2 - 1) = \boxed{x^4 - 1}$$

$$(x-1)(x+1) = x^2 - 1^2 = x^2 - 1$$

<p>РАЗЛИКА КВАДРАТА $(I + II) \cdot (I - II) = I^2 - II^2$</p>
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$$2) (x-3) \cdot (x^2 + 9) \cdot (x+3) = (x^2 - 9)(x^2 + 9) = \boxed{x^4 - 81}$$

$$x^2 - 3^2 = x^2 - 9$$

$$3) (x^3 - 5) \cdot (x^6 + 25) \cdot (x^3 + 5) = (x^6 - 25)(x^6 + 25) = \boxed{x^{12} - 625}$$

$$(x^3)^2 - 5^2 = x^6 - 25$$

$$4) (3x^2 + 7) \cdot (9x^4 + 49) \cdot (3x^2 - 7) = (9x^4 - 49)(9x^4 + 49) = \boxed{81x^8 - 2401}$$

$$(3x^2)^2 - 7^2 = 9x^4 - 49$$

4. Упрости изразе:

$$1) (x+2)^2 - 4 = (x^2 + 2 \cdot x \cdot 2 + 2^2) - 4 = x^2 + 4x + 4 - 4 = \boxed{x^2 + 4x}$$

$$2) 5 - (3x-1)^2 = 5 - ((3x)^2 - 2 \cdot 3x \cdot 1 + 1^2) = 5 - (9x^2 - 6x + 1) = 5 - 9x^2 + 6x - 1 = \boxed{-9x^2 + 6x + 4}$$

$$3) (4x-1)^2 - x(2x-5) = ((4x)^2 - 2 \cdot 4x \cdot 1 + 1^2) - 2x^2 + 5x = 16x^2 - 8x + 1 - 2x^2 + 5x = \boxed{14x^2 - 3x + 1}$$

$$4) (2x+3)^2 + 4x^2 - 2x(x+2) = (4x^2 + 12x + 9) + 4x^2 - 2x^2 - 4x = \boxed{6x^2 + 8x + 9}$$

$$5) 3x(7-3x) - (3x+1)^2 = 21x - 9x^2 - (9x^2 + 6x + 1) = 21x - 9x^2 - 9x^2 - 6x - 1 = \boxed{-18x^2 + 15x - 1}$$

$$6) -4x(4-x) - 2x(x+2) = -16x + 4x^2 - 2x^2 - 4x = \boxed{2x^2 - 20x}$$

5. Прво упрости израз а затим израчунај његову бројевну вредност:

$$1) (2x-3y)^2 - (2x-3y) \cdot (2x+3y) \quad \text{за} \quad x = \frac{1}{2}, \quad y = \frac{1}{3};$$

$$\begin{aligned}
 & (4x^2 - 12xy + 9y^2 - (4x^2 - 9y^2)) = \\
 & = \cancel{4x^2} - 12xy + 9y^2 - \cancel{4x^2} + 9y^2 = 18y^2 - 12xy = 18 \cdot \\
 & 18 \cdot \left(\frac{1}{3}\right)^2 - 12 \cdot \frac{1}{2} \cdot \frac{1}{3} = 18 \cdot \frac{1}{9} - 2 = 2 - 2 = \boxed{0}
 \end{aligned}$$

$$2) (4x-3y)^2 - 2x \cdot (8x-5y) \quad \text{за} \quad x = \frac{1}{4}, \quad y = -2;$$

$$\begin{aligned}
 & \cancel{16x^2} - 24xy + 9y^2 - \cancel{16x^2} + 10xy = -14xy + 9y^2 \\
 & -14 \cdot \frac{1}{4} \cdot (-2) + 9 \cdot (-2)^2 = 7 + 9 \cdot 4 = 7 + 36 = \boxed{43}
 \end{aligned}$$

$$3) (x+y)^2 + (x-y)^2 + (x+y) \cdot (x+y); \quad \text{за} \quad x=2, \quad y=-3;$$

$$\begin{aligned}
 & \cancel{x^2} + \cancel{2xy} + \cancel{y^2} + \cancel{x^2} - \cancel{2xy} + \cancel{y^2} + \cancel{x^2} + \cancel{xy} + \cancel{xy} + \cancel{y^2} \\
 & = 3x^2 + y^2 \\
 & 3 \cdot 2^2 + (-3)^2 = 3 \cdot 4 + 9 = 12 + 9 = \boxed{21}
 \end{aligned}$$

$$4) \left(\frac{1}{2}x-2y\right)^2 - \left(\frac{1}{2}x-2y\right) \cdot \left(\frac{1}{2}x+2y\right) \quad \text{за} \quad x = \sqrt{2}, \quad y = \frac{1}{\sqrt{2}};$$

$$\begin{aligned}
 & \left(\frac{1}{2}x\right)^2 - 2 \cdot \frac{1}{2}x \cdot 2y + (2y)^2 - \left(\left(\frac{1}{2}x\right)^2 - (2y)^2\right) = \\
 & = \frac{1}{4}x^2 - 2xy + 4y^2 - \left(\frac{1}{4}x^2 - 4y^2\right) = \\
 & = \cancel{\frac{1}{4}x^2} - 2xy + 4y^2 - \cancel{\frac{1}{4}x^2} + 4y^2 = 8y^2 - 2xy \\
 & 8 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 - 2 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = 8 \cdot \frac{1}{2} - 2 = 4 - 2 = \boxed{2}
 \end{aligned}$$

6. Дати су биноми $A=1-2a$ и $B=-3a+2$. Одреди:

$$1) A+B = (1-2a) + (-3a+2) = 1-2a-3a+2 = \boxed{-5a+3}$$

$$2) A-B = (1-2a) - (-3a+2) = 1-2a+3a-2 = \boxed{a-1}$$

$$3) A^2 = (1-2a)^2 = \boxed{1-4a+4a^2}$$

$$4) B^2 = (-3a+2)^2 = (-3a)^2 + 2 \cdot (-3a) \cdot 2 + 2^2 = \boxed{9a^2-12a+4}$$

$$5) A \cdot B = (1-2a)(-3a+2) = -3a+2+6a^2-4a = \boxed{6a^2-7a+2}$$

7. Дати су биноми $A=x+3$ и $B=x-3$. Одреди:

$$1) 4A-3B = 4(x+3) - 3(x-3) = 4x+12-3x+9 = \boxed{x+21}$$

$$2) A \cdot B = (x+3) \cdot (x-3) = \boxed{x^2-9}$$

$$3) A^2 + B^2 = (x+3)^2 + (x-3)^2 = x^2+6x+9 + x^2-6x+9 = \boxed{2x^2+18}$$

$$4) A^2 - B^2 = (x+3)^2 - (x-3)^2 = x^2+6x+9 - (x^2-6x+9) = \cancel{x^2+6x+9} - \cancel{x^2-6x+9} = \boxed{12x}$$

$$5) (A-B)^2 = ((x+3)-(x-3))^2 = (x+3-x+3)^2 = 6^2 = \boxed{36}$$

$$6) (A+B)^2 = (x+3+x-3)^2 = (2x)^2 = \boxed{4x^2}$$

8. Дати су полиноми $A=3x-1$, $B=3x+1$ и $C=x+1$. Израчунај:

$$1) A^2 + B^2 = (3x-1)^2 + (3x+1)^2 = 9x^2 - 6x + 1 + 9x^2 + 6x + 1 = \boxed{18x^2 + 2}$$

$$2) (A-B)^2 \cdot C = ((3x-1) - (3x+1))^2 \cdot (x+1) = (3x-1-3x-1)^2 \cdot (x+1) = (-2)^2 \cdot (x+1) = 4 \cdot (x+1) = \boxed{4x+4}$$

$$3) A^2 + B^2 - 18 \cdot C^2 = (3x-1)^2 + (3x+1)^2 - 18(x+1)^2 = 9x^2 - 6x + 1 + 9x^2 + 6x + 1 - 18(x^2 + 2x + 1) = 18x^2 + 2 - 18x^2 - 36x - 18 = \boxed{-36x - 16}$$

$$4) A^2 - 9 \cdot (B-C)^2 = (3x-1)^2 - 9 \cdot ((3x+1) - (x+1))^2 = 9x^2 - 6x + 1 - 9(3x+1-x-1)^2 = 9x^2 - 6x + 1 - 9(2x)^2 = 9x^2 - 6x + 1 - 9 \cdot 4x^2 = 9x^2 - 6x + 1 - 36x^2 = \boxed{-27x^2 - 6x + 1}$$

9. Реши једначине:

$$1) (x-2)^2 - x^2 = 24;$$

$$\cancel{x^2} - 4x + 4 - \cancel{x^2} = 24$$

$$-4x + 4 = 24$$

$$-4x = 24 - 4$$

$$-4x = 20$$

$$x = \frac{20}{-4}$$

$$\boxed{x = -5}$$

$$2) x^2 - (x-5)^2 = -20;$$

$$\cancel{x^2} - (\cancel{x^2} - 10x + 25) = -20$$

$$\cancel{x^2} - \cancel{x^2} + 10x - 25 = -20$$

$$10x - 25 = -20$$

$$10x = -20 + 25$$

$$10x = 5$$

$$x = \frac{5}{10}$$

$$\boxed{x = \frac{1}{2}}$$

$$3) (2x-3)^2 - 4x^2 - 3 = 1;$$

$$\cancel{4x^2} - 12x + 9 - \cancel{4x^2} - 3 = 1$$

$$-12x + 6 = 1$$

$$-12x = 1 - 6$$

$$-12x = -5$$

$$x = \frac{-5}{-12}$$

$$\boxed{x = \frac{5}{12}}$$

$$4) (2x-3)^2 - 4x^2 - 1 = 1;$$

$$\cancel{4x^2} - 12x + 9 - \cancel{4x^2} - 1 = 1$$

$$4x^2 - 12x + 8 = 1$$

$$-12x = 1 - 8$$

$$-12x = -7$$

$$\boxed{x = \frac{7}{12}}$$

$$5) 4 \cdot (x+3) \cdot (5-4x) + (4x-5)^2 = 4;$$

$$4 \cdot (5x - 4x^2 + 15 - 12x) + (16x^2 - 40x + 25) = 4$$

$$\underline{20x - 16x^2 + 60 - 48x + 16x^2 - 40x + 25 = 4}$$

$$-68x + 85 = 4$$

$$-68x = 4 - 85$$

$$-68x = -81$$

$$x = \frac{-81}{-68}$$

$$\boxed{x = \frac{81}{68}}$$

$$6) 8x^2 - (4x-3) \cdot (2x+3) = 9;$$

$$8x^2 - (8x^2 + 12x - 6x - 9) = 9$$

$$\cancel{8x^2} - \cancel{8x^2} - 12x + 6x + 9 = 9$$

$$-6x + 9 = 9$$

$$-6x = 9 - 9$$

$$-6x = 0$$

$$x = \frac{0}{-6}$$

$$\boxed{x = 0}$$

$$7) 10 - 2 \cdot (2x-1)^2 - 8x \cdot (3-x) = 0;$$

$$10 - 2 \cdot (4x^2 - 4x + 1) - 24x + 8x^2 = 0$$

$$\cancel{10} - \cancel{8x^2} + 8x - 2 - 24x + \cancel{8x^2} = 0$$

$$-16x + 8 = 0$$

$$-16x = 0 - 8$$

$$-16x = -8$$

$$x = \frac{-8}{-16}$$

$$\boxed{x = \frac{1}{2}}$$

$$8) 2 \cdot (2x+1)^2 - 8 \cdot (x-2) \cdot (x+2) = 2;$$

$$2 \cdot (4x^2 + 4x + 1) - 8(x^2 - 4) = 2$$

$$\cancel{8x^2} + 8x + 2 - \cancel{8x^2} + 32 = 2$$

$$8x + 34 = 2$$

$$8x = 2 - 34$$

$$8x = -32$$

$$x = \frac{-32}{8}$$

$$\boxed{x = -4}$$

$$9) (4x-1)^2 - 2(x+1)(8x-3) = -11;$$

$$16x^2 - 8x + 1 - 2(8x^2 - 3x + 8x - 3) = -11$$

$$\cancel{16x^2} - 8x + 1 - \cancel{16x^2} + 6x - 16x + 6 = -11$$

$$-18x + 7 = -11$$

$$-18x = -11 - 7$$

$$-18x = -18$$

$$x = \frac{-18}{-18}$$

$$\boxed{x = 1}$$

$$10) (2y-1) \cdot (y+3) - 2(2-y)^2 = 15.$$

$$2y^2 + 6y - y - 3 - 2(4 - 4y + y^2) = 15$$

$$\cancel{2y^2} + 5y - 3 - 8 + 8y - \cancel{2y^2} = 15$$

$$13y - 11 = 15$$

$$13y = 15 + 11$$

$$13y = 26$$

$$y = \frac{26}{13}$$

$$\boxed{y = 2}$$